

2-categorical Cohen-Montgomery duality for category pseudo-actions

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Throughout this talk \mathbb{k} denotes a commutative ring. We first note that a group pseudo-action of a group G on a category \mathcal{C} defined by Deligne [2] and Drinfeld–Gelaki–Nikshych–Ostrik [3] is nothing but a pseudofunctor from G as a groupoid with a single object $*$ to the 2-category \mathbf{CAT} of categories sending $*$ to \mathcal{C} . Thus if \mathcal{C} is a small \mathbb{k} -category, then it is just a pseudofunctor $X: G \rightarrow \mathbb{k}\text{-Cat}$ with $X(*) = \mathcal{C}$, where $\mathbb{k}\text{-Cat}$ is the 2-category of small \mathbb{k} -categories. We denote by $G\text{-Cat}$ the 2-category of small \mathbb{k} -categories with G -pseudo-actions, and by $G\text{-GrCat}$ the 2-category of small G -graded \mathbb{k} -categories. By generalizing the main result in [1] it is possible to show that a 2-functor $?/G: G\text{-Cat} \rightarrow G\text{-GrCat}$ defined by extending the orbit category construction is a 2-equivalence with a 2-quasi-inverse $?#G: G\text{-GrCat} \rightarrow G\text{-Cat}$ defined by extending the smash product. By replacing the group G by a small category I we extend this result. Denote by $\text{Pfun}(I, \mathbb{k}\text{-Cat})$ the 2-category of pseudofunctors $I \rightarrow \mathbb{k}\text{-Cat}$, and by $I\text{-GrCat}$ the 2-category of small I -graded \mathbb{k} -categories. Then we can generalize the Grothendieck construction to a 2-functor $\text{Gr}: \text{Pfun}(I, \mathbb{k}\text{-Cat}) \rightarrow I\text{-GrCat}$ and define the smash product 2-functor $?#I: I\text{-GrCat} \rightarrow \text{Pfun}(I, \mathbb{k}\text{-Cat})$ in such a way that they are 2-quasi-inverses to each other. Of course, if $I = G$ then we have $\text{Gr} = ?/G$ and $?#I = ?#G$.

1. Asashiba, H.: A generalization of Gabriel’s Galois covering functors II: 2-categorical Cohen-Montgomery duality, *Applied Categorical Structures* **25** (2017), no. 2, 155–186.
2. Deligne, P.: Action du groupe des tresses sur une catégorie, *Invent. Math.* **128** (1997), 159–175.
3. Drinfeld, V., Gelaki, S., Nikshych, D., and Ostrik, V.: On braided fusion categories, I, *Sel. Math. New Ser.* **16** (2010), 1–119.