

Degeneration of zero in Triangulated Categories

Alexander Zimmermann

Université de Picardie, Amiens, FRANCE

alexander.zimmermann@u-picardie.fr

In this talk I report on joint work with Manuel Saorín, building on earlier joint work with Bernt Tore Jensen and Xiuping Su.

For an algebra A over an algebraically closed field k the possible A -module structures on k^d form an affine variety, on which a group acts, the orbits parameterising isomorphism classes. A module M degenerates to N if N belongs to the Zariski closure of the orbit of M . G. Zwara and Ch. Riedtmann characterised this concept in purely algebraic terms by means of the existence of certain short exact sequences.

Replacing short exact sequences by distinguished triangles we get a degeneration concept in triangulated categories. We develop a geometric interpretation of this concept generalising work of Y. Yoshino, and obtain in some cases that this is equivalent with the algebraic one.

It was known for a long time that this triangulated version allows the zero object to degenerate, contrary to the module case. We study this phenomenon systematically, for the geometric version as well as for the algebraic one. In the geometric case degeneration of zero is closely linked to torsion phenomena of parameter spaces and in the algebraic case we show that degeneration of zero is intrinsic in the whole concept. Further, we detail the relation of the class of objects having zero image in the Grothendieck group with those being degenerations of zero.