

Comparing localisations of commutative noetherian rings

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Given a commutative noetherian ring R , Gabriel and Neeman have shown that both hereditary torsion classes of $\text{Mod}(R)$ and smashing subcategories of $D(R)$ correspond bijectively to specialisation closed subsets of $\text{Spec}(R)$. This assignment relies on the notion of support. Hereditary torsion classes and smashing subcategories are intrinsically related to certain localisations of $\text{Mod}(R)$ and $D(R)$, respectively. Neeman has furthermore shown that every smashing subcategory is generated by a set of compact objects in $D(R)$, proving the telescope conjecture for such rings.

In this talk we focus on the localisation functors arising from a flat ring epimorphism $R \rightarrow S$, namely the tensor product with S and its derived functor. The kernels of these functors are, respectively, a hereditary torsion class in $\text{Mod}(R)$ and a smashing subcategory in $D(R)$. Using this fact, we show that these kernels can be characterised in terms of support and the vanishing of local cohomology. This then allows us to prove that, for a large class of commutative rings, these smashing subcategories are generated by two-term compact objects and that, therefore, flat ring epimorphisms can be understood as universal localisations in the sense of Cohn and Schofield. This is joint work with L. Angeleri Hügel, F. Marks, J. V. Svrtov'ic and R. Takahashi.