Spanier-Whitehead categories of resolving subcategories and comparison with singularity categories

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Let $\mathcal{A}$ be an abelian category with enough projective objects, and let $\mathcal{X}$ be a quasi-resolving subcategory of $\mathcal{A}$. Then the stable category $\underline{\mathcal{X}}$ modulo projectives with syzygy functor forms a left triangulated category. Inspired by a well-known construction in algebraic topology, there is a triangulated category $\mathsf{SW}(\mathcal{X})$ associated to $\underline{\mathcal{X}}$, which called the Spanier-Whitehead category of $\underline{\mathcal{X}}$. By a result of A. Beligiannis the singularity category $\mathsf{D}_{\mathsf{sg}}(\mathcal{A})$ of $\mathcal{A}$ is triangle equivalent to $\mathsf{SW}(\mathcal{A})$. We therefore obtain a fully faithful triangle functor

$$\theta_{\mathcal{X}} : \mathsf{SW}(\mathcal{X}) \longrightarrow \mathsf{D}_{\mathsf{sg}}(\mathcal{A}),$$

sending each object $X[i]$ to the complex concentrated in degree $-i$.

In this talk, we investigate the affinity of the Spanier-Whitehead category $\mathsf{SW}(\mathcal{X})$ with the singularity category $\mathsf{D}_{\mathsf{sg}}(\mathcal{A})$ and prove that $\theta_{\mathcal{X}}$ is dense if and only if the bounded derived category $\mathsf{D}^{\mathsf{b}}(\mathcal{A})$ is generated by the objects in $\mathcal{X}$. Applying this result, we can characterize some properties of commutative noetherian rings R. We show that based on resolving subcategories $\mathcal{X}$ of the category $\mathsf{mod}R$, the density of the fully faithful functor $\theta_{\mathcal{X}}$ reflects being an isolated singularity, Cohen-Macaulayness and Gorensteiness of the base ring R. We should emphasize that the importance of this result comes from the fact that for non-Gorenstein rings, not much is known about their singularity categories.