Tilting modules and tilting torsion pairs

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joint work with: Francesco Mattiello, Alberto Tonolo

This communication is about a forthcoming joint paper with Francesco Mattiello and Alberto Tonolo.

Let A be a ring and A-Mod the category of left A-modules. The notion of tilting module has been axiomatised in 1979 by Brenner and Butler [BB], generalising that of progenerator for modules of projective dimension 1. Such a *classical 1-tilting* module T determines a torsion pair $(KE_0(T), KE_1(T))$ on the category of modules, where

$$KE_i(T) = \{M \in A\text{-Mod} : \mathrm{Ext}^j(T, M) = 0 \ \forall j \neq i\}, \quad i = 0, 1.$$

Therefore, each module M admits a filtration with two factors belonging to $KE_i(T)$ for $i = 0, 1$.

Miyashita [M] extended the tilting notion for modules of projective dimension $n \geq 1$. A *classical n-tilting* module T naturally gives rise to $n+1$ classes of modules in A-Mod, called *Miyashita classes*:

$$KE_i(T) = \{M \in A\text{-Mod} : \mathrm{Ext}^j(T, M) = 0 \ \forall j \neq i\}, \quad i = 0, 1, ..., n.$$

For $n > 1$, the Miyashita classes are too small in order to filter every left A-module.

Working on the derived category $\mathcal{D}(A)$, the Miyashita classes can be equivalently described as

$$KE_i(T) = \{M \in A\text{-Mod} : \mathrm{Hom}_{\mathcal{D}(A)}(T, M[j]) = 0 \ \forall j \neq i\}, \quad i = 0, 1, ..., n.$$

Consider the larger classes

$$\mathcal{KE}_i(T) = \{M^\bullet \in \mathcal{D}(A) : \mathrm{Hom}_{\mathcal{D}(A)}(T, M^\bullet[j]) = 0 \ \forall j \neq i\}, \quad i = 0, 1, ..., n.$$

Generalising [FMT], we provide, for a not necessarily finitely generated n-tilting module T, a decomposition of any module in terms of objects in these classes. This decomposition generalises the one found for $n = 1$: the Miyashita classes can indeed be regarded as the piece of these new classes visible in the category of modules.

For $i = 1, ..., n$, the class $\mathcal{KE}_i(T)$ is the $(-i)$-shift of the heart of the t-structure $\mathcal{T}$ associated to the tilting module T. The decomposition of any modules is obtained using n torsion pairs in the hearts of as many t-structures linking the natural t-structure to the t-structure $\mathcal{T}$ in $\mathcal{D}(A)$.

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