On Homotopy Category of Generalized U-Complexes

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Davvaz and Shabbani-Solt introdruced the notion of chain U-complex as a genera-lization of chain complex. A sequence of R-modules and R-homomorphism in abelian category $\mathcal{A}$, $\rightarrow X_{n+1} \overset{d_{n+1}}{\longrightarrow} X_n \overset{d_n}{\longrightarrow} X_{n-1} \longrightarrow$, is called chain U-complex if it satisfies: (1) $d_n d_{n+1}(X_{n+1}) \subseteq U_{n-1}$ and (2) $U_n \subseteq d_{n+1}(X_{n+1})$. A morphism between two chain U-complexes is a chain map that preserves each of submodules U_n. Two chain maps are U-homotopic if there exists a homotopy map that preserves each U_n. We define the homotopy category of U-complexes analougously with homotopy category of complexes, and denoted as $\mathbf{K}(\mathcal{A}, U)$. Suppose $f : (X, U^X, d^X) \rightarrow (Y, U^Y, d^Y)$ be a morphism in $\mathbf{K}(\mathcal{A}, U)$.We define the mapping cone of f by $M(f)_n = X_{n-1} + Y_n, U_n^{M(f)} = U^X \oplus U^Y$ and $d_n^{M(f)} = \begin{pmatrix} -d_{n-1}^X & 0 \\ f_{n-1} & d_n^Y \end{pmatrix}$. Generally it is not an object in $\mathbf{K}(\mathcal{A}, U)$, because it does not satisfy (2). Here, we propose a generalization of chain U-complex by changing (2) with $(2')\, d_n(U_n) \subseteq U_{n-1}$. Then we define the homotopy category of generalized U-complexes and study its properties.