

Definable coaisles over rings of weak global dimension at most one

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In the setting of the unbounded derived category $\mathcal{D}(R)$ of a ring R of weak global dimension at most one we consider t-structures with a definable coaisle. The t-structures among these which are stable (that is, the t-structures which consist of a pair of triangulated subcategories) are precisely the ones associated to a smashing localization of the derived category. In this way, our present results generalize those of [BS] to the non-stable case.

We show that any definable subcategory of $\mathcal{D}(R)$ is determined on the cohomology and we characterize the definable coaisles by certain chains of subcategories of the module category. As in the stable case [BS], we then confine to the commutative setting, and give a full classification of definable coaisles in the local case, that is, over valuation domains. It turns out that unlike in the stable case of smashing localizing subcategories, the definable coaisles do not always arise from homological ring epimorphisms. We show that this phenomenon is closely related to the (non-)density in the total order of the “admissible systems” of intervals associated to a definable coaisle.

We also consider a non-stable version of the telescope conjecture for t-structures and give a ring-theoretic characterization of the commutative rings of weak global dimension at most one for which it is satisfied. Also, we prove that all definable coaisles are compactly generated in case the ring is (not necessarily commutative) von Neumann regular.

[BS] Silvana Bazzoni and Jan Šťovíček, Smashing localizations of rings of weak global dimension one. *Advances in Mathematics* 305 (2017): 351-401.